A note on the emergence of cosmic space in modified gravities

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Abstract

Intrigued by the holographic principle, Padmanabhan recently proposed a novel idea saying that our cosmic space is emergent as cosmic time progresses. In particular, the expansion rate of the universe is related to the difference between the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside. In this note, we generalize this interesting paradigm to brane world, scalar-tensor gravity and f(R) theory, and find that in the cosmological setting the Friedmann equations can be successfully derived respectively.

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I. INTRODUCTION

The tight link between a gravitational system and a thermodynamical system was discovered in 1970s[1]. A thermodynamical description of the Einstein equation was originally proposed by Jacobson [2]. In this scenario the Einstein equation becomes an equation of state and can be derived from a fundamental thermodynamical relation, namely Clausius relation $\delta Q = TdS$, which connects heat, entropy and temperature for all the local Rindler causal horizons. Recently, this scenario has been demonstrated in many gravity theories and cosmological models[3–15]. In this context, the dynamical equations of the gravitational field can be derived from the Clausius relation. Furthermore, the viewpoint of gravity being not a fundamental interaction has been developed in the paper by Verlinde [16]. Gravity is explained as an entropic force which is caused by the change of the information associated with the position of matter. In [17], Padmanabhan also argued that the equipartition law of energy for the horizon degrees of freedom can lead to the Newton's law of gravity through the thermodynamical relation $S = \frac{E}{2T}$, where E is the active gravitational mass producing the gravitational acceleration in the spacetime [18].

Very recently, Padmanabhan presented a novel way to view the cosmic space as an emergent phenomenon in a cosmological setting[19]. The main idea is attributing the expansion of our universe to the difference between the surface degree of freedom on holographic horizon and the bulk degrees of freedom in bulk. In this paradigm the dynamical equation of a FRW universe can be successfully derived. After that, this setup has been generalized to the cosmology in Gauss-Bonnet gravity and more general Lovelock gravity in[20, 21]. Further discussions on its applications in the cosmology by Padmanabhan can be found in[22].

We may briefly summarize the idea proposed by Padmanabhan as follows. For a pure de Sitter universe with Hubble parameter H, the holographic principle can be described by the relation

$$N_{sur} = N_{bulk}, \tag{1}$$

where N_{sur} denotes the number of the degrees of freedom on the holographic screen with Hubble radius $1/H$, namely $N_{sur} = \frac{4\pi}{L_p^2 H^2}$ with $L_p^2 = G$, while $N_{bulk} = \frac{2|E|}{T}$ is the number of the degrees of freedom in bulk, where $|E| = |\mathcal{M}| = |\rho+3p|V$ is the Komar mass. The horizon temperature is determined by $T = \frac{H}{2\pi}$. One called the above equation as the holographic equipartition. Since the real world is not purely but asymptotically de Sitter, then one may propose that the expansion rate of the cosmic volume is related to the difference of these

two degrees of freedom as

$$\frac{dV}{dt} = L_p^2(N_{sur} - N_{bulk}). \tag{2}$$

As shown in [19], Eq. (2) shows that it is very necessary for the existence of cosmological constant to drive the expansion of the universe toward holographic equipartition. Substituting the cosmic volume $V = \frac{4\pi}{3H^3}$ and the number of the degrees of freedom into above equation, one obtained the standard dynamical Friedmann equation in general relativity,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{3}$$

In addition, making use of the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{4}$$

and integrating Eq.(3), one can obtain the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho, \tag{5}$$

where the integration constant has been set to zero.

In this note, we will testify this interesting proposal in other gravitational theories. We intend to generalize this paradigm to the brane world, scalar-tensor gravity and f(R) gravity. The key issue is how to find the correct result for the Komar mass in these modified gravity theories since it plays an essential role in this route. Our strategy is to introduce a total effective energy-momentum tensor such that the equations of motion have the same form as the one in standard general relativity. As a result we can derive the Komar mass in a similar way as that in general relativity.

Our paper is organized as follows. In Section 2, we treat the expansion of the cosmic space as an emergent process and derive the Friedmann equations in the context of brane world, while in section 3 and 4, we intend to derive the Friedmann equations in scalar-tensor theory and f(R) gravity respectively. Our conclusion and discussion is given in the last section. We shall set the constants $\hbar = c = k_B = 1$ in this paper.

II. DYNAMICAL FRIEDMANN EQUATIONS AS AN EMERGENCE OF THE COSMIC SPACE IN BRANE WORLD

In this section, we consider how this paradigm can deduce the equation of motion of our universe in the context of the brane world. First of all, we introduce the metric of the

background as

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{6}$$

where $k = 1, 0, -1$ corresponds to a closed, a flat and an open universe respectively. Without loss of generality, we may only consider the spatially flat FRW universe without cosmological constant embedded in a Randall-Sundrum model. In a brane world, the key difference is to determine Komar mass in term of T_{ab}. In Ref.[24] the Komar mass for a brane world has been derived as

$$\begin{aligned}\mathcal{M} &= \int_v dV(\rho + 3P + \frac{2\rho^2}{\lambda} + \frac{3\rho P}{\lambda}) \\ &= \frac{4\pi}{3H^3}(\rho + 3P + \frac{2\rho^2}{\lambda} + \frac{3\rho P}{\lambda}),\end{aligned} \tag{7}$$

where we have identified the Hubble horizon as the radius of the holographic screen and λ is the brane tension which is tuned with the five dimensional cosmological constant by $\lambda \sim -\frac{\Lambda_5}{8\pi G_4}$. Furthermore it is worthy to note that we have used the total effective energy-momentum tensor for brane world. Substituting Eq.(7)into Eq.(2), we have

$$\begin{aligned}\frac{dV}{dt} &= L_p^2(\frac{A}{G} - \frac{-2\mathcal{M}}{T}) \\ &= \frac{4\pi}{H^2} + \frac{16\pi^2 G}{3H^4}(\rho + 3P + \frac{2\rho^2}{\lambda} + \frac{3\rho P}{\lambda}).\end{aligned} \tag{8}$$

Simplifying the above equation leads to the standard dynamical Friedmann equation in brane cosmology

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P + \frac{2\rho^2}{\lambda} + \frac{3\rho P}{\lambda}). \tag{9}$$

Moreover, using the continuity equation $\dot{\rho} + 3H(\rho + P) = 0$, one can easily derive the other Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho(1 + \frac{\rho}{2\lambda}). \tag{10}$$

III. DYNAMICAL FRIEDMANN EQUATIONS AS AN EMERGENCE OF THE COSMIC SPACE IN SCALAR-TENSOR GRAVITY

Now we in turn consider the emergence of the cosmic space in scalar-tensor gravity. The equation of motion in scalar-tensor gravity can be written as [25]

$$\begin{aligned}G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \\ &= \frac{8\pi G}{f(\phi)}[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 - g_{\mu\nu}V(\phi) - g_{\mu\nu}\nabla^2 f(\phi) + \nabla_\mu\nabla_\nu f(\phi) + T^{(m)}_{\mu\nu}],\end{aligned} \tag{11}$$

and

$$\nabla^2\phi - V'(\phi) + \frac{1}{2}f'(\phi)R = 0 \tag{12}$$

where $G_{\mu\nu}$ and $T^{(m)}_{\mu\nu}$ denote the Einstein tensor and the energy-momentum tensor of matter respectively, while R is the Ricci curvature scalar. $f(\phi)$ is a generic function of a scalar field ϕ. Here we only consider the flat universe. For convenience, we can define an energy-momentum tensor for the scalar field

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 - g_{\mu\nu}V(\phi) - g_{\mu\nu}\nabla^2 f(\phi) + \nabla_\mu\nabla_\nu f(\phi). \tag{13}$$

Moreover, we assume the energy-momentum tensor $T^{(\phi)}_{\mu\nu}$ has the same form as that of a perfect fluid,

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}. \tag{14}$$

It is worthwhile to stress that such an assumption is compatible with the hypothesis of homogeneity and isotropy of the unverse. Then correspondingly we can find the energy density as well as the pressure of the scalar field as

$$\rho^{(\phi)} = \frac{1}{2}(\dot\phi)^2 + V - 3H\dot f, \tag{15}$$

$$P^{(\phi)} = \frac{1}{2}(\dot\phi)^2 - V + 2H\dot f + \ddot f. \tag{16}$$

Taking the ordinary matter into account, we can further define an total effective energy-momentum tensor as

$$\begin{aligned} T^{(t)}_{\mu\nu} &= [\frac{\rho^{(m)} + P^{(m)} + \rho^{(\phi)} + P^{(\phi)}}{f(\phi)}]u_\mu u_\nu + [\frac{P^{(m)} + P^{(\phi)}}{f(\phi)}]g_{\mu\nu} \\ &= [\frac{\rho^{(m)} + P^{(m)} + \frac{1}{2}(\dot\phi)^2 + V - 3H\dot f + \frac{1}{2}(\dot\phi)^2 - V + 2H\dot f + \ddot f}{f(\phi)}]u_\mu u_\nu \\ &\quad + [\frac{P^{(m)} + \frac{1}{2}(\dot\phi)^2 - V + 2H\dot f + \ddot f}{f(\phi)}]g_{\mu\nu}. \end{aligned} \tag{17}$$

As a result, the equation of motion in scalar-tensor theory can be rewritten as a compact form

$$G_{\mu\nu} = 8\pi G T^{(t)}_{\mu\nu}. \tag{18}$$

Now we turn to consider deriving the dynamic Friedmann equations in scalar-tensor theory from an emergent point of view. Due to the above equation we should still adopt the assumptions $N_{sur} = \frac{A}{G} = \frac{4\pi}{GH^2}$ and $T = \frac{H}{2\pi}$ in scalar-tensor theory [1]. Next the key step is to

[1] Theoretically one expects that such assumptions should be supported by the thermodynamics of black holes in scalar-tensor gravity. Such subtleties has previously been analyzed in [25].

find out the effective Komar mass in scalar-tensor theory. Since the equation of motion for scalar-tensor gravity has the same form as the one in standard general relativity, we argue that the Komar mass in scalar-tensor cosmology can be still evaluated as

$$\begin{aligned} \mathcal{M} &= 2\int_v dV[T^{(t)}_{\mu\nu} - \frac{1}{2}T^{(t)}g_{\mu\nu}]u^\mu u^\nu \\ &= V[\frac{\rho^{(m)}+3P^{(m)}}{f(\phi)} + \frac{\frac{1}{2}(\dot{\phi})^2 + V - 3H\dot{f} + 3(\frac{1}{2}(\dot{\phi})^2 - V + 2H\dot{f} + \ddot{f})}{f(\phi)}]. \end{aligned} \tag{19}$$

As a consequence, the expansion rate of the universe in scalar-tensor cosmology can be obtained from Eq.(2) as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\frac{\rho^{(m)}+3P^{(m)}}{f(\phi)} + \frac{2(\dot{\phi})^2 - 2V + 3H\dot{f} + 3\ddot{f}}{f(\phi)}). \tag{20}$$

Using the effective energy density in Eq.(15) and pressure density in Eq.(16), we can write this equation into the familiar form

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\frac{\rho^{(m)}+3P^{(m)}}{f(\phi)} + \frac{\rho^{(\phi)}+3P^{(\phi)}}{f(\phi)}). \tag{21}$$

Moreover, due to the Bianchi identity, $\nabla^\mu G_{\mu\nu} = 0$, we have the conservational relation of the total effective energy-momentum tensor $\nabla^\mu T^{(t)}_{\mu\nu} = 0$ which gives out the continuity equation as

$$\frac{d}{dt}(\frac{\rho^{(\phi)}+\rho^{(m)}}{f(\phi)}) + 3H(\frac{\rho^{(m)}+P^{(m)}+\rho^{(\phi)}+P^{(\phi)}}{f(\phi)}) = 0. \tag{22}$$

Making the use of the above continuity equation and integrating Eq.(20), we can finally obtain the other Friedmann equation as

$$H^2 = \frac{8\pi G}{3}[\frac{\rho^{(m)}+\rho^{(\phi)}}{f(\phi)}]. \tag{23}$$

Here we have also set the integration constant to vanish.

IV. DYNAMICAL FRIEDMANN EQUATIONS AS AN EMERGENCE OF THE COSMIC SPACE IN $f(R)$ THEORY

In last section along the same tactic we consider the emergence of the cosmic space in $f(R)$ theory concisely. In literature [25–30], the equation of motion in f(R) gravity is given by

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \\ &= 8\pi G[\frac{T^{(m)}_{\mu\nu}}{f'(R)} + \frac{f(R)g_{\mu\nu}}{16\pi G f'(R)} - \frac{Rf'(R)g_{\mu\nu}}{16\pi G f'(R)} + \frac{1}{8\pi G f'(R)}(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)f'(R)], \end{aligned} \tag{24}$$

where $T^{(m)}_{\mu\nu}$ is the energy-momentum tensor of matter and $f' \equiv \frac{df(R)}{dR}$. In the cosmological setting f(R) is an arbitrary function of Ricci scalar curvature R with $R = 6(\dot{H} + 2H^2 + \frac{k}{a^2})$. Here we only consider the flat universe with $k = 0$. For simplicity, we may define a total effective energy-momentum tensor for both ordinary matter and effective curvature fluid, which is read as [25]

$$T^{(t)}_{\mu\nu} = [\frac{\rho^{(m)}}{f'} + \frac{P^{(m)}}{f'} + \rho^{(c)} + P^{(c)}]u_\mu u_\nu + [\frac{P^{(m)}}{f'} + P^{(c)}]g_{\mu\nu} \tag{25}$$

where $\rho^{(c)} = \frac{Rf'-f-6H\dot{R}f''}{16\pi Gf'}$ and $P^{(c)} = \frac{\dot{R}^2 f'''+2H\dot{R}f''+\ddot{R}f''-\frac{1}{2}(Rf'-f)}{8\pi Gf'}$. As a result, the equation of motion in f(R) theory can also be written as a compact form as Eq.(18). In a parallel way we obtain the Komar mass in f(R) theory and then plug it into Eq.(2) such that the dynamical equation in $f(R)$ cosmology can be obtained as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\frac{\rho^{(m)} + 3P^{(m)}}{f'} + \rho^{(c)} + 3P^{(c)}), \tag{26}$$

where we have used the assumptions $N_{sur} = \frac{A}{G} = \frac{4\pi}{GH^2}$ and $T = \frac{H}{2\pi}$ in $f(R)$ theory. Moreover, due to the Bianchi identity, we get the continuity equation as

$$\frac{d}{dt}(\rho^{(c)} + \frac{\rho^{(m)}}{f'}) + 3H(\frac{\rho^{(m)} + P^{(m)}}{f'} + \rho^{(c)} + P^{(c)}) = 0. \tag{27}$$

Making use of the above continuity equation and integrating Eq.(26), we can finally obtain the other Friedmann equation as

$$H^2 = \frac{8\pi G}{3}[\frac{\rho^{(m)}}{f'} + \rho^{(c)}]. \tag{28}$$

Here we have also set the integration constant to vanish. The above cosmological equations in f(R) theory are consistent with the results in literature[25][2]. In the end of this section we would like to present a remark on the the area-entropy relation in both scalar-tensor gravity theory as well as the $f(R)$ theory. Once all the contributions due to the scalar field or the curvature fluid are absorbed into the total effective energy-momentum tensor, we may think of them as some effective matter such that Einstein field equations take the same form as the standard one. Then one would expect that the area-entropy relation should be preserved as the usual one, namely $S = \frac{A}{4G}$, though the total effective energy-momentum tensor may provide important corrections to the size and shape of the horizon.

[2] After we have completed this manuscript we noticed the latest paper [23] which overlaps with our content in this section, while in [23] they employed a different area-entropy relation such that the Friedmann dynamical equation is modified. For us it seems not transparent to derive the corresponding Friedmann kinematic equation from that modified dynamical equation.

V. CONCLUSIONS AND DISCUSSIONS

In this note we have applied the idea of treating the cosmic space as an emergent process to brane cosmology, scalar-tensor cosmology and f(R) gravity. We found the corresponding cosmological equations in these theories can be obtained such that the holographic nature of this idea has been further testified in a more general setting.

We expect this paradigm can be further improved to be consistent with the thermodynamics of black holes when the corrections due to the quantum effects of gravity are taken into account. For instance, it is well known that such quantum gravity effects may contribute a logarithmic correction to the entropy of black holes. Correspondingly, it is reasonable to conjecture that the number of degrees of freedom on the holographic screen is not exactly proportional to the one fourth of the area of the horizon, but containing extra corrections like a logarithmic term. How to take such extra contributions into account and consider their impacts on the modification of Friedmann equations should be an interesting issue in future.

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